A LAMINAR BOUNDARY LAYER WITH DISPERSED IMPURITY ENTRAINED FROM A PLATE SURFACE

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A model of a gas-dispersed laminar boundary layer (LBL) on a semiinfinite plate is proposed. Unlike the majority of published works, in which the dispersed impurity permeates through an external LBL boundary, in the present work the source of the dispersed phase is the particles entrained by a flow from a surface in the flow with a small longitudinal velocity. In the LBL, the particles get into a region where the longitudinal velocity of the carrier-medium is large; therefore the Safman lift force is directed upwards and tends to carry them out of the LBL. As a result of increasing the dispersed impurity velocity, their concentration rapidly falls at a distance from the surface. Downward from the flow, the flux of particles entrained from the surface decreases to zero, the LBL becomes a one-phase flow.

Recently a gas-dispersed laminar boundary layer on a plate has been investigated in a number of works (see [1-3] and bibliography therein). In [1], the structure of a dust-laden LBL is studied on an impermeable plate at $Re_d \to 0$. In [2], the plate is permeable for a gas but not for particles. In [3], it is shown that at $Re_d \sim 1$ it is necessary to take account of lift forces acting on dispersed particles; a dispersed impurity deposits on a plate. In the present work, consideration is given to an LBL with a dispersed impurity entrained from a plate surface. Interest in such a flow is awakened by the fact that it may serve, to some extent, as a model of wind-sand flow formation [4]. In [4], $Re_d > 1$ but lift forces are neglected, which is valid only at $Re_d << 1$. The dynamics of low-inertia particles in an LBL ($Re_d \to 0$) essentially differs from inertial ones as a result of lift forces acting on the latter [3]. In [4], the interphase interaction force in the momentum equation is represented as $F_x = k(u_p - u)$, k = const, but by meaning $k = \rho_p/\text{Stk}$ and at a variable particle concentration (and it is variable in [4], the profile $\rho_p(y)$ is given) it cannot be constant.

We consider an LBL on a semiinfinite plate into which a dispersed impurity, entrained from a surface with a flow past it, may be brought. Assuming that the flow is weakly dust-laden, direct interaction between particles may be neglected; the particle Reynolds force is $Re_d \sim 1$, then the Magnus lift force owing to rotation of particles may be neglected as well, and we take account only of the Safman lift force, acting on particles in the LBL [3].

The system of equations of a gas-dispersed LBL with the above assumptions differs from [3] only by a gravity force taken into account:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{f}{\text{Stk}}(u - u_p)\rho_p, \tag{3}$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{f}{\text{Stk}}(u - u_p), \tag{4}$$

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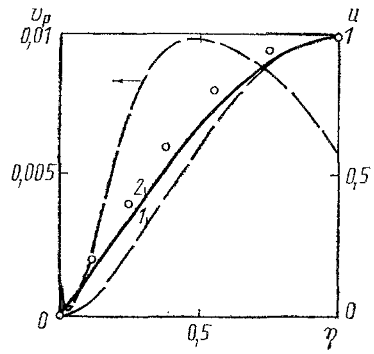

Fig. 1

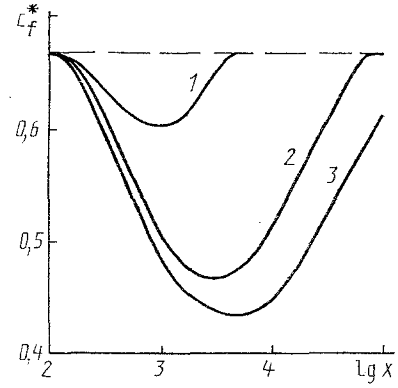

Fig. 2

Fig. 1. Dimensionless velocity profiles of carrier (the solid curve) and dispersed (the dashed curve) phases at $\mathrm{Re_d} = 1$, $\mathrm{Re_x} = 2 \cdot 10^4$, $\mathrm{Fr_d} = 1.5 \cdot 10^4$; points, the longitudinal gas velocity at $\mathrm{Re_x} \to \infty$ (the self-similar Blasius solution).

Fig. 2. Local drag of the plate: 1) $\mathrm{Re_d} = 1$, $\mathrm{Fr_d} = 0.5 \cdot 10^4$; 2) 1 and $1.5 \cdot 10^4$; 3) 3 and $1.5 \cdot 10^4$.

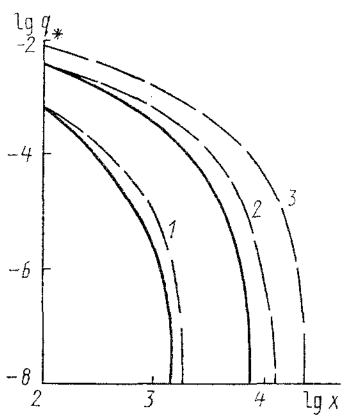

Fig. 3

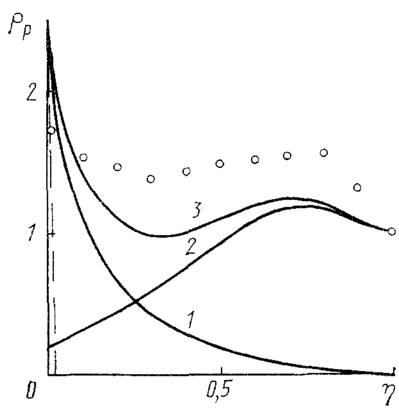

Fig. 4

Fig. 3. Dimensionless dispersed impurity flux entrained from the surface (notation is the same as in Fig. 2).

Fig. 4. Concentration profiles of the dispersed impurity: 1) entrained from the surface; 2) falling onto the surface; 3) total concentration; points) experimental data [4]; dashed line) the calculations disregarding the Safman force.

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{f}{\mathrm{Stk}} (v - v_p) - G + (u - u_p) a \sqrt{\tau}\,. \tag{5}$$

The boundary conditions of the set of equations for a carrier-medium were given analogously [1, 3]:

$$y = 0\colon\ u_w = v_w = 0;\quad y \to \infty\colon\ u_e = 1.$$

Near the front cover of the plate, the one-phase LBL Blasius profiles were given for a gas (without dispersed impurity)

$$\rho_{p0}(y) = 0,\ u_{p0}(y) = 0,\ v_{p0}(y) = 0.$$

To prescribe boundary conditions for the dispersed phase on a plate, we shall use the data from [5], in which the expression is experimentally obtained for a dispersed impurity flux entrained from a plate surface with flow past the plate. In terms of the accepted designations, this expression has the form

$$q \equiv \rho_{pw} v_{pw} = b_1 (b_2 \sqrt{\tau_w} - b_3)^3,$$
$$b_1 = 2{,}1 \cdot 10^{-5} / (\lambda^{3/2}\, \mathrm{Fr}_d^{1/2}), \tag{6}$$
$$b_2 = \mathrm{Re}_d (\lambda\, \mathrm{Fr}_d / \mathrm{Re}_d^2)^{0,43}, \quad b_3 = 0{,}1585.$$

In [5], carriages with solid particle samples were installed flush with the plate surface, with the flow past the plate. Determination was made of the entrainment rate of rolled sandy soil particles at a density R_p° = 2720 kg/m^3. Preliminarily, the soil was screened to single out sufficiently narrow particle fractions with a mean diameter of 140 μm to 2 mm. The incoming flow velocity was varied from 3 to 9 m/sec. Froude numbers of particles in [5] were $\mathrm{Fr}_d = (0.2\text{-}5.8)\cdot 10^4$. The distance from the front edge of the plate to the working section amounted to 0.06-0.5 m, which corresponded to $\mathrm{Re}_x = (0.1\text{-}3)\cdot 10^5$. This Re_x range points to the fact that the experiments [5] are conducted in an LBL; therefore, the formula (6) obtained from the above experimental results may be used to determine boundary conditions in the present work.

Obviously, near the surface $\rho_{pw} \sim \alpha/\lambda$, where $\alpha < 1$, the quantity v_{pw} is to be determined by dividing q by ρ_{pw} and $u_{pw} \sim 0$, since for the particles having an appreciable longitudinal velocity on the surface, the left force, causing their entrainment from the surface, drastically decreases.

The set of Eqs. (1)-(5) with the given boundary conditions has been solved by a finite-difference method similar to [1]. Some calculation results are shown in Figs. 1-4. Calculations are performed at $\lambda = 0.4 \cdot 10^{-3}$.

Figure 1 demonstrates the calculated velocity profiles. With the dispersed impurity entrained from the surface, the longitudinal velocity profile of a carrier-medium (curve 2) is less filled than the Blasius profile of a one-phase LBL (points). This is attributed to the fact that the longitudinal velocity of particles entrained from the surface is small near the plate (curve 1) and the momentum is transferred from the carrier-to-dispersed phase. At $\mathrm{Re}_x \to \infty$, the impurity is not entrained any more from the surface (see Fig. 3); the LBL becomes the one-phase layer with the Blasius velocity profile. Curve 3 is the longitudinal velocity profile of the dispersed impurity. Its value near the wall drastically decreases due to the gravity force; then, at a distance from the plate, it increases owing to the Safman lift force, attains its maximum and again decreases with decreasing $u - u_p$ and τ entering into the above force term [the last term in Eq. (5)]. If we neglect the lift force in Eq. (5), then v_p, because of gravity, quickly decreases to zero, i.e., a particle, having risen to an insignificant height above the plate, will fall onto it again.

Thus, calculation without the Safman force qualitatively distorts the result: the dispersed impurity is not entrained from the gas-dispersed LBL via an external boundary, but it remains in a negligibly thin layer near the plate surface (see the dashed line in Fig. 4).

Figure 2 represents local drag variation along the plate

$$c_f \equiv \nu \frac{\partial U}{\partial Y} \Big/ \frac{1}{2} U_e^2 = 2\tau_w, \quad c_f^* = c_f \sqrt{x}. \tag{7}$$

As is known, in the one-phase LBL, $c_f^* = 0.664$ = const (the dashed line in Fig. 2). The presence of the dispersed impurity near the surface, with the longitudinal velocity smaller than the carrier-medium velocity, causes a decrease of c_f^*. The larger Re_d and Fr_d, the lower and farther along x the minimum drag c_f^* comes before the impurity flux entrained from the surface decreases to zero (Fig. 3).

Figure 3 shows the variation of the dispersed impurity flux entrained from the surface, calculated by formula (6). The dashed lines are the calculated results at $\tau_w \sqrt{x} = 0.332$, i.e., without an account of the dispersed impurity effect on the carrier-medium velocity; the solid lines are calculation with regard for this influence. Since the quantity τ, as compared to a one-phase LBL, decreases when the incoming flow has no dispersed impurity, the solid curve tends to zero more rapidly than the dashed curve. With decreasing the Froude Fr_d and Reynolds Re_d numbers, the magnitude of the dispersed impurity flux decreases which is consistent with experimental data [5]. Downward from the flow, at $\tau_w < (b_3/b_2)^2$ the impurity is not entrained from the surface. The earlier entrained impurity permeates through the external LBL boundary and the boundary layer becomes a one-phase layer.

Figure 4 represents the dispersed impurity concentration profiles. The concentration of the impurity entrained from the surface (curve 1) drastically decreases at a distance from the plate, which is explained by an increase of its velocity in the flow. Compare the calculated and experimental concentrations [4]. It should be taken into account that in [4] the dispersed impurity is not only entrained from the surface but also permeates through the external LBL boundary, i.e., the flow passing around the surface is a gas-dispersed flow, as in [1-3]. The dynamics of these particles differs from that of particles entrained from the surface. The dispersed particles from the external flow have a transverse velocity, owing to the Safman force, directed toward the plate [3]. If we assume that these particles settle down onto the surface, then in conformity with [3] their concentration profile has the form of curve 2 in Fig. 4. The total concentration of the dispersed phase is obtained, apparently, by adding the concentrations of two particle fluxes. Curve 3, obtained in such a manner, qualitatively agrees with experimental data [4] (points), and has a specific wavelike form. A quantitative difference between the calculated and experimental data is likely associated with the necessity of taking into account the interaction of particle fluxes incident onto the surface and entrained from it.

Thus, the model suggested is consistent with the physical sense of the problem and the results calculated do not contradict experimental data. This proves that the model may be used as a model of wind-sand flow formation. An increase of the model universality is related to consideration of the interaction of dispersed flows, namely, entrained from the surface and incoming from the external flow, as well as the latter with the surface.

NOTATION

$x = XU_e/\nu \equiv Re_x$; $y = YU_e/\nu \equiv Re_y$; X,Y, longitudinal and transverse coordinates; δ, LBL thickness; $\eta = Y/\delta$; ν, kinematic viscosity of a carrier-medium; $u = U/U_c$, $v = V/U_e$, dimensionless longitudinal and transverse velocities; $\tau = \partial u/\partial y$; R_p, distributed density of the dispersed impurity; $\rho_p = R_p/R^o$, $\lambda = R^o/R_p^o$, R^o, R_p^o, real phase densities; $Re_d = U_e d/\nu$, Reynolds number; d, particle diameter; f, function describing the difference of the particle resistance law from the Stokes formula; $Stk = Re_d^2/(18\lambda)$, Stokes number; $q = \rho_{pw}v_{pw}$, dimensionless flux of the entrained dispersed impurity; $q_* = q\sqrt{x}$; $G = Fr_d^{-1}Re_d^{-1}$; $Fr_d = U_e^2/(gd)$, Froude number; g, free fall acceleration; $a = 3.075\lambda/Re_d$; c_f, local drag of the plate. Indices: w, plate; e, external flow; 0, initial cross section; p, dispersed phase.

LITERATURE CITED

1. A. N. Osiptsov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4, 48-54 (1980).
2. A. M. Grishin and V. I. Zabarin, Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 54-61 (1987).
3. V. A. Naumov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6, 176-178 (1988).
4. M. M. Ismailov, Dokl. Akad. Nauk Uzbek. SSR, No. 3, 17-18 (1982).
5. B. R. Toshov, Mechanics of One and Two-Phase Media [in Russian], Tashkent (1988), pp. 23-36.